INFORMATION THEORY

EEEN 464 – DIGITAL COMMUNICATION

Friday, 06 March 2026

WHERE WE ARE IN THE SYLLABUS

Course Content:

Signal digitization: Pulse Amplitude Modulation (PAM), sampling theorems and sampling circuits, Pulse code modulation (PCM). Quantization and signal conditioning: Uniform and non-uniform quantization; companding methods; vocoders; signal-to- quantization noise ratio. Waveform coding: Pulse transmission, PCM, Pulse-shaping; Delta modulation; adaptive delta modulation; Differential Pulse Code Modulation (DPCM), M-ary encoding. Digital Modulation: Amplitude shift keying (ASK), Frequency Shift Keying (FSK), Phase Shift Keying (PSK), Quadrature Amplitude Modulation (QAM) and Differential Phase Shift Keying (DPSK). Signal recovery in ASK, FSK and PSK; Gaussian Minimum Shift Keying (GMSK); Performance comparison. Information theory: information sources, entropy, channel capacity; Source Coding; entropy coding. Error control: Error control coding techniques; Transmission errors; Error detection methods; intersymbol interference and the eye pattern; Linear block codes; Cyclic codes; convolution codes. Multiplexing: Frequency division multiplex (FDM), Time Division Multiplexing (TDM), plesiochronous digital hierarchy (PDH). Spread spectrum communication: Direct sequence and frequency hopping methods; synchronization, spreading codes and their generation. Data transmission: Local data transmission protocals (Ethernet, token ring); Modems; high Asymmetric Digital subscriber line (ADSL); Very–high Speed Digital subscriber line (VDSL), integrated services digital network (ISDN).

INTRODUCTION TO INFORMATION THEORY

1. The purpose of communication is to carry information bearing baseband signals from one place to another over a communication channel.
2. **Information theory** is a branch of probability theory which deals with mathematical modelling and analysis of communication systems rather than the physical channels.
3. **Information theory** was invented by scientists studying the statistical structure of electronic communication systems.
4. Information theory attempts to provide answers to questions like what is the amount of information carried?

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IEEE INFORMATION THEORY SOCIETY

About the IEEE Information Theory Society

1. **IEEE Information Theory Society** is the premier professional society dedicated to the advancement of the mathematical underpinnings of information technology for the benefit of humanity.
2. Information theory encompasses the processing, transmission, storage, and use of information, and the foundations of the communication process.

Home | IEEE Information Theory Society (itsoc.org)

WHY IS INFORMATION THEORY IMPORTANT?

1. **Information theory is important because it provides the mathematical framework for understanding and optimizing data communication, storage, and processing**, impacting various fields like communications, data compression, and machine learning.

2. **In communication engineering,** information theory is fundamental in designing communications systems, enabling efficient encoding and transmission of data with minimal errors and high data rates.

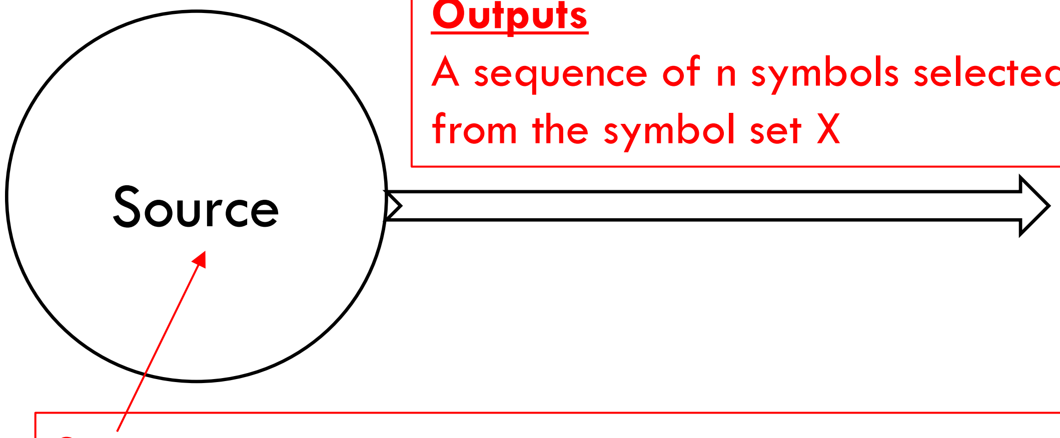
3. **In Data Compression, information theory provides the theoretical limits of data compression**, allowing for the development of efficient algorithms that minimize storage space and transmission costs.

4. **In machine Learning, many probabilistic approaches are based on measures derived from information theory**, including entropy, mutual information, and relative entropy.

INFORMATION SOURCE

1. An information Source may be viewed as an object that produces random events.
2. An information Source can be analogue or discrete (digital)
3. A discrete information source has a finite number of symbols as possible outputs.

Outputs
A sequence of n symbols selected from the symbol set X

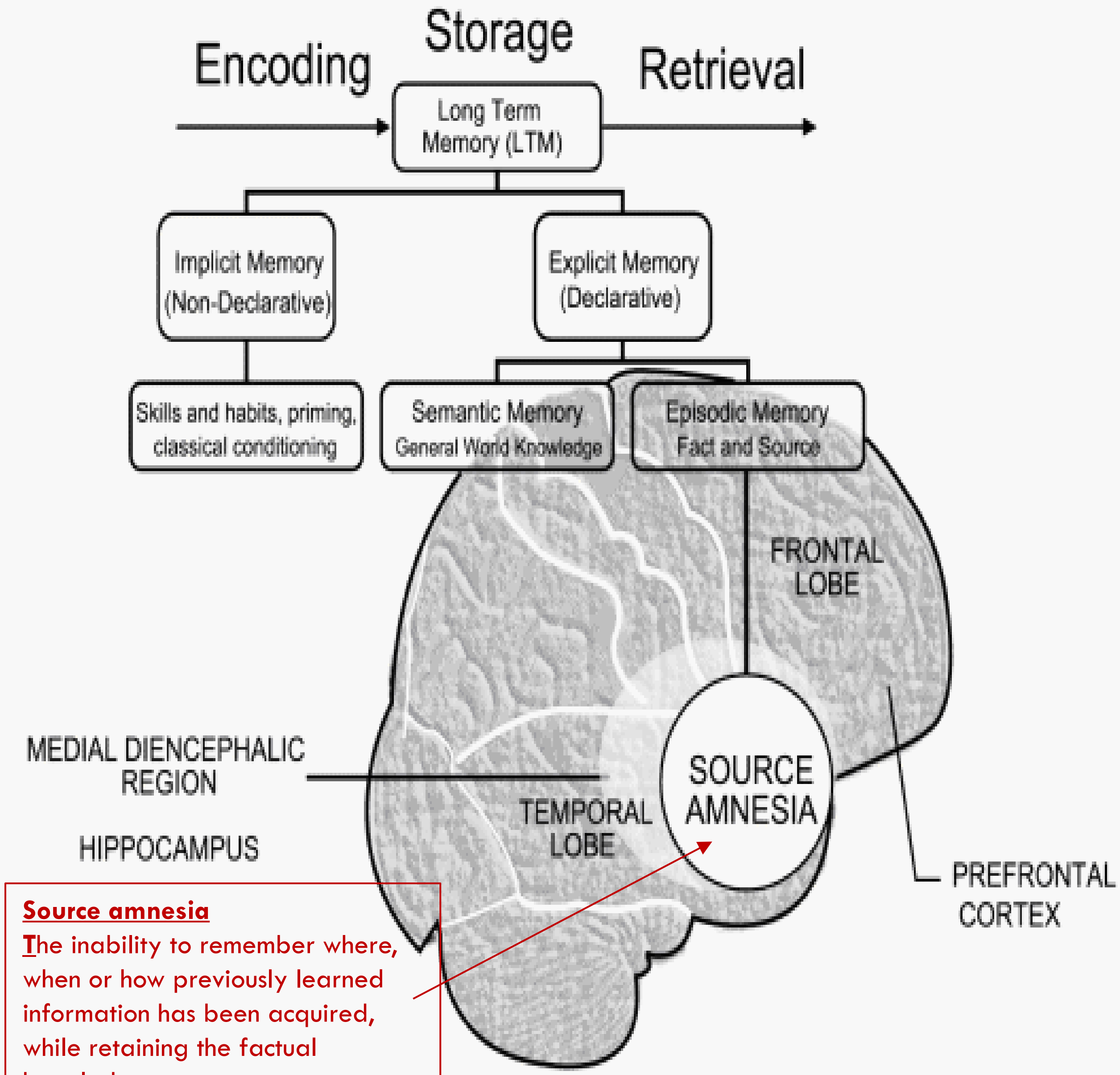


Source
A random generator with a set, $X = \{x_1, x_2, x_3, \ldots . x_N\}$ capable of generating N symbols.

CLASSIFICATION OF INFORMATION SOURCES

1. Information Sources can be classified as having memory or being memory less.
   a) **Memory Source:** The current symbol depends on previous symbols
   b) **Memory-less source:** Each symbol produced is independent of previous symbols.

2. **Discrete Memory-less Source (DMS)** consists of a discrete set of letters or alphabetic symbols.



Source amnesia
The inability to remember where, when or how previously learned information has been acquired, while retaining the factual knowledge

AMOUNT OF INFORMATION?

Which of the following has more information?

1. The EEEN 464 class has 35 students
2. The EEEN 464 class is in Room LBB 109
3. The Continuous assessment test will be in the fourth week of the semester
4. The lecturers are on strike

The least likely event contains maximum information.

AMOUNT OF INFORMATION IN A DISCRETE MEMORY-LESS SYSTEM (DMS)

1. The amount of information in an event is related to its uncertainty.
   a) Messages conveying a high probability of occurrence convey relatively little information.
   b) If an event is certain, i.e probability of occurrence is one, then it conveys zero information.

2. **A mathematical measure** of information should therefore satisfy the following axioms:
   a) Information should be proportional to the uncertainty of an outcome
   b) Information contained in independent outcomes should add.

INFORMATION CONTENT OF A SYMBOL (1)

Assume a Discrete Memoryless Source (DMS) denoted by X and having an output alphabet $\{x_1, x_2, x_3, \ldots x_n\}$

The information content of a symbol x_i is defined as:

$$I(x_i) = log_b \frac{1}{P(x_i)} = -log_b P(x_i)$$

Where $P(x_i)$ *is the probability of occurance of symbol* x_i

Characteristics of Information Content:

a) $I(x_i) = 0$ for $P(x_i) = 1$

b) $I(x_i) \geq 0$

c) $I(x_i) > I(x_j)$ if $P(x_i) < P(x_j)$

d) $I(x_i, x_j) = I(x_i) + I(x_j)$ if x_i and x_j are independent

INFORMATION CONTENT OF A SYMBOL (2)

1. $I(x_i) = log_b \frac{1}{P(x_i)} = -log_b P(x_i)$

   a) If the base, b=2 then units are bits or Hartleys
   b) If the base, b=10 then the units are Decits
   c) If the base b= e then the units are nat (natural)

2. We can convert between different units by using the equation:

$$log_2 = \frac{In(a)}{In(2)}$$

INFORMATION CONTENT – WORKED EXAMPLE 1

1. A Source produces four possible symbols during each observation interval having probability $P(x_1)=\frac{1}{2}$, $P(x_2)=\frac{1}{4}$, $P(x_3)=P(x_4)=\frac{1}{8}$

Calculate the information content of each symbol in bits

Answer:

$I(x_i) = -log_2(P(x_i))$

$$I(x_1) = -\,log_2\left(\frac{1}{2}\right) = 1bit$$

$$I(x_2) = -\,log_2\left(\frac{1}{4}\right) = 2bits$$

$$I(x_3) = -\,log_2\left(\frac{1}{8}\right) = 3bits$$

$$I(x_4) = -\,log_2\left(\frac{1}{8}\right) = 3bits$$

INFORMATION CONTENT – WORKED EXAMPLE 2

2. Calculate the amount of information of the binary digits which occur with equal likelihood in a PCM system.

Answer:

There are two levels in a PCM systems, i.e $x_0 = 0$, and $x_1 = 1$

If the levels occur with equal probability as given then

$P(x_i)$ for level 0 $= \frac{1}{2}$

$P(x_i)$ for level 1 $= \frac{1}{2}$

Therefore:

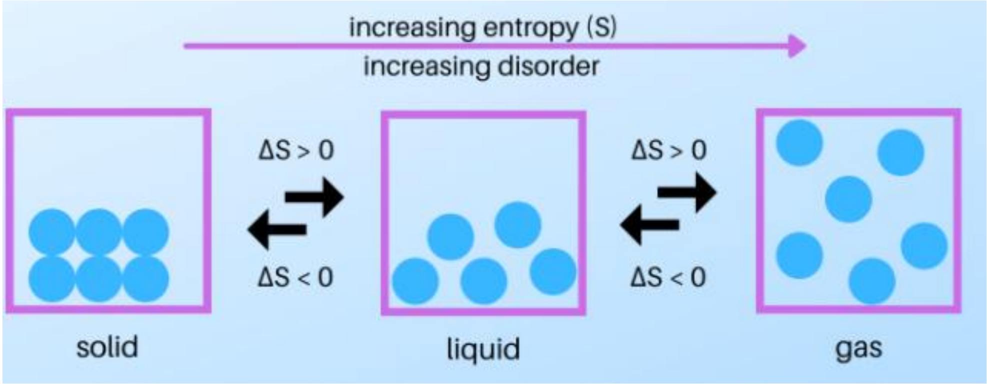
$$I(x_0) = -log_2\left(\frac{1}{2}\right) = 1\ bit\ for\ level\ 0$$

$$I(x_1) = -log_2\left(\frac{1}{2}\right) = 1\ bit\ for\ level\ 1$$

WHAT IS ENTROPY?

General definition

Entropy is a scientific concept as well as a measurable physical property that is most commonly associated with a state of disorder, randomness, or uncertainty.



Entropy in Information Theory

Entropy of a random variable is the average level of "information“ ("surprise", or "uncertainty")inherent to the variable's possible outcomes.

ENTROPY OF INFORMATION SYSTEM

In practice, an information source produces a long sequence of random symbols.

There is therefore more interest in the average information(or Entropy) rather than the information content of a single symbol.

Assumptions for computing average information:

1. The source is stationary thus enabling the probabilities to remain constant with time.
2. Successive symbols are statistically independent and are generated at a constant rate, r

ENTROPY-MATHEMATICAL EXPRESSION

The mathematical expression of Entropy is:

$$H(X) = E\{I(x_i)\} = \sum_{i=1}^{m} P(x_i)I(x_i)$$

$$H(X) = -\sum_{i=1}^{m} P(x_i)log_2P(x_i) \qquad bits/symbol$$

H(X) is referred to as entropy of source X and measures the average information content of a source symbol.

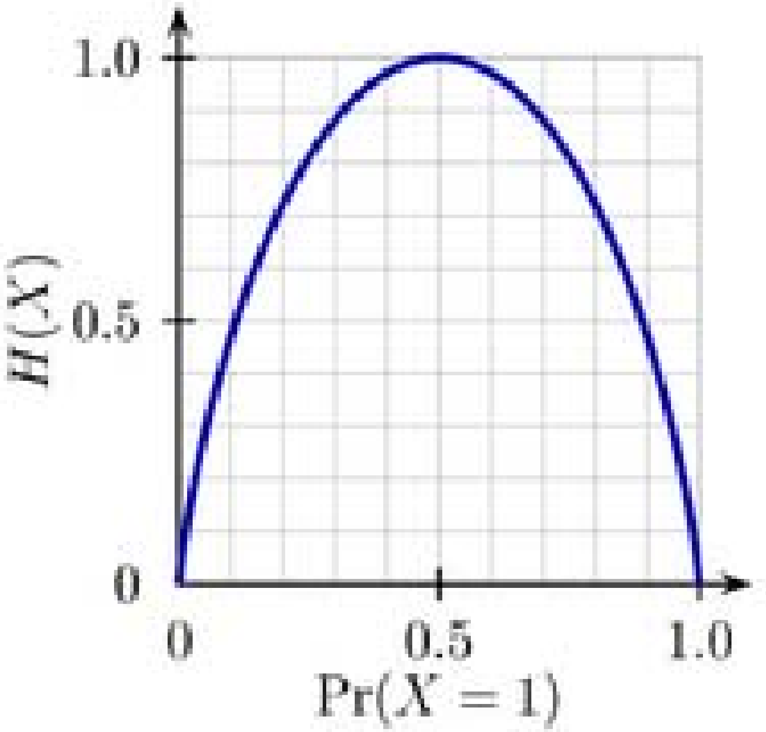
Entropy H(X) can be considered as the average amount of uncertainty within source X that can be resolved by the use of it's alphabet.

ENTROPY OF BINARY SOURCE

If a binary source X generates independent symbol 0 and 1 with equal probability, then the source entropy is given by:

$$H(X) = -\sum_{i=1}^{m} P(x_i) log_2 P(x_i)$$

$$H(X) = -\frac{1}{2} log_2\left(\frac{1}{2}\right) - (-\frac{1}{2} log_2\left(\frac{1}{2}\right)) = 1 \text{ bit/symbol}$$



(a) If $\Pr(X \neq \frac{1}{2})$ then H(X) takes the values indicated in the graph above.

LOWER AND UPPER BONDS ON ENTROPY FOR M SYMBOLS

Source entropy H(X) satisfies the following relation:

$$0 \leq H(X) \leq log_2 m$$

Where m is the number of symbols in the source alphabet.

1. H(X) = 0 when only one symbol has probability $P(x_i)=1$ while $P(x_j)=0$ for $j \neq i$.

2. H(X)=1 when $P(x_i) = \frac{1}{m}$ *for all i*

ENTROPY FOR INDEPENTENT SOURCES

If x_i and x_j are independent, then

$$P(x_i, x_j) = P(x_i)P(x_j)$$

Therefore

$$I(x_i x_j) = log_b \frac{1}{P(x_i x_j)} = log_b \frac{1}{P(x_i)P(x_j)} = log_b \frac{1}{P(x_i)} + log_b \frac{1}{P(x_j)}$$

$$= I(x_i) + I(x_j)$$

INFORMATION RATE

If a source X emits symbols at a rate r, then the information rate is given by r times the entropy, or

$$R = rH(X)\ bits/Sec$$

ENTROPY EXAMPLE 1

A discrete memory less source X has four symbols $\{x_1, x_2, x_3, x_4\}$with probabilities $p(x_1)=0.4$, $p(x_2)=0.3$, $p(x_3)=0.2$, $p(x_4)=0.1$

Calculate the entropy, H(X).

Solution:

$$H(X) = -\sum_{i=1}^{4} P(x_i) log_2(P(x_i))$$

$$H(X) = -0.4log_2(0.4) - 0.3log_2(0.3) - 0.2log_2(0.2) - 0.1log_2(0.1)$$

$$= 1.85 \text{ bits/Symbol}$$

INFORMATION RATE EXAMPLE 2

A high resolution black and white TV picture consists of 2 x 10^6 picture elements (pixels) and 16 different brightness levels. Pictures are repeated at the rate of 32 frames per second.

Assuming that all picture elements are independent and all levels have equal likelihood of occurrence, calculate the average rate of information conveyed by the TV picture source.

Solution:

Let m = Number of symbols = Number of brightness levels = 16

$$H(X) = -\sum_{i=1}^{m} P(x_i) log_2(P(x_i) = -\sum_{i=1}^{16} \frac{1}{16} log_2 \frac{1}{16} = 4\ bits/pixel$$

Pixel rate, r = No of pixels x No of frames

= 2 x 10^6 x 32 = 64 x 10^6 pixels/sec

The average rate of information is therefore

$$R = rH(x) = 64\ x\ 10^6 \ \text{x}\ 4 = 256\ \text{Mb/s}$$

INFORMATION RATE EXAMPLE 3

An analogue transducer signal is band-limited to f_m = 30 Hz and is sampled at Nyquist rate before being quantized at 4 levels.

If each level represents one symbol and the probabilities of occurrence of these four levels are $P(x_1)=P(x_2) = 1/8$ and $P(x_3)=P(x_4) = 3/8$, derive the expression for the information rate.

Solution:

Let Entropy be H(X)

$$H(X) = -P(x_1)log_2(x_1) - P(x_2)log_2(x_2) - P(x_3)log_2(x_3) - P(x_4)log_2(x_4)$$

$$H(X) = \frac{1}{8}log_2 8 + \frac{1}{8}log_2 8 + \frac{3}{8}log_2(\frac{8}{3}) + \frac{3}{8}log_2(\frac{8}{3})$$ =1.8 bits/sec

At Nyquist rate, r = $2f_m$ = 60 therefore

Information rate, R = rH(x) = $2f_m$ x 1.8 = $3.6f_m$ = 3.6x 60 = 216 bits/sec

ENTROPY AND INFORMATION RATE EXAMPLE

Assume that an analogue signal is band-limited to 10 KHz and is sampled at the Nyquist rate and quantized in 8 levels with probabilities ¼, 1/5, 1/5, 1/10,1/10, 1/20, 1/20, 1/20 respectively. Calculate the information rate.

Solution:

Entropy, $H(X) = -\sum_{i=1}^{4} P(x_i) log_2(P(x_i)$

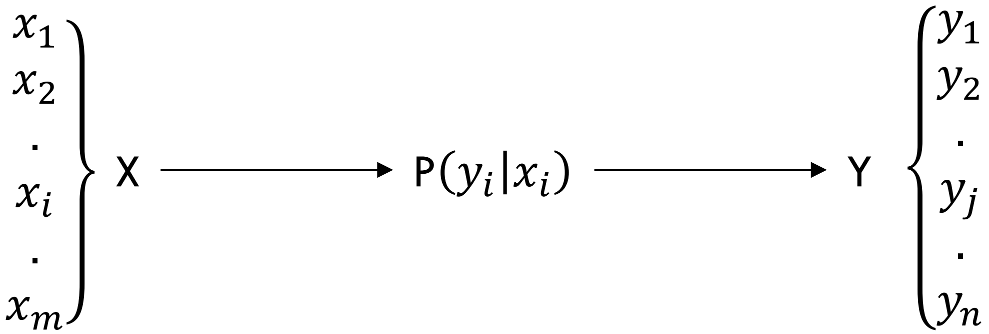
$H(X) = \frac{1}{4} log_2 4 + \frac{1}{5} log_2 5 + \frac{1}{5} log_2 5 + \frac{1}{10} log_2 10 + \frac{1}{10} log_2 10 + \frac{1}{20} log_2 20 + \frac{1}{20} log_2 20 + \frac{1}{20} log_2 20 =$ 2.84 bits/sec

The Nyquist rate is $2f_m$ = 2X 10,000 = 20,000 samples (symbols) per second

Information rate, R = rH(X) = 20,000 x 2.84 = 56,800 bits/sec

DISCRETE MEMORYLESS CHANNEL

1. In terms of information theory, a communication channel is a path or medium through which symbols flow from source to destination.
2. A discrete Memory less channel is a statistical model of a channel through which m symbols are input and n outputs are produced.
3. The a priori probabilities of the input symbols are assumed to be known.
4. Each possible input/output path is represented by a conditional probability $P(y_j|x_i)$



CHANNEL MATRIX

A channel is completely specified by a set of transitional probabilities.

$$[P(Y|X)] = \begin{bmatrix} P(y_1|x_1) & P(y_2|x_1) & \cdots & P(y_n|x_1) \\ P(y_1|x_2) & P(y_2|x_2) & \cdots & P(y_n|x_2) \\ \cdots & \cdots & \cdots & \cdots \\ P(y_1|x_m) & P(y_2|x_m) & \cdots & P(y_n|x_m) \end{bmatrix}$$

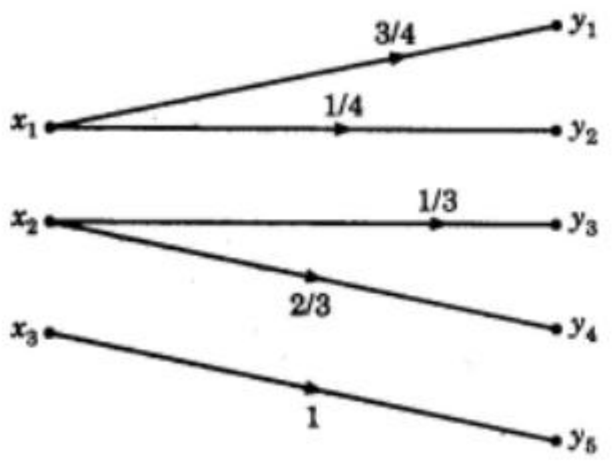
Since each input to the channel results in some output, each row of the matrix must sum to unity, i.e

$$\sum_{j=1}^{n} P(y_j : x_i) = 1 \quad for\ all\ i$$

LOSSLESS CHANNEL

A lossless channel has only one non-zero element in each column.

$$[P(Y|X)] = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

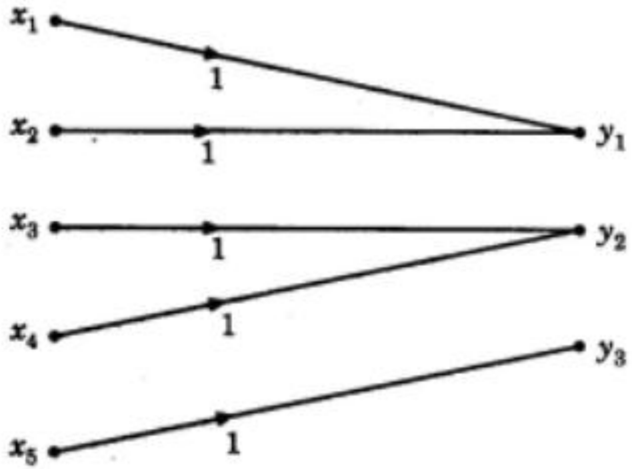


No information is lost in transmission

DETERMINISTIC CHANNEL

1. A deterministic channel is the one with only one none zero element in a row.

$$[P(Y|X)] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

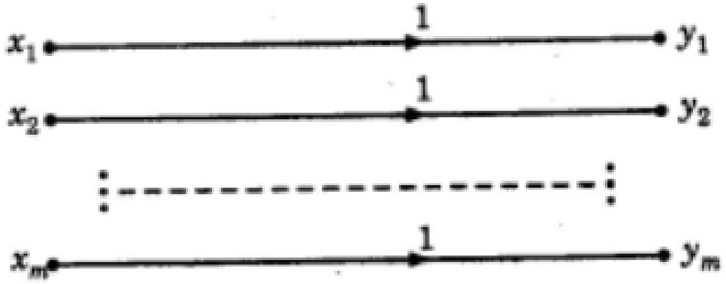


2. Since each row has one element, it follows that the element must be unity.
3. When a given source symbol is sent, it is clear which output symbol will be received. Hence the name - deterministic.

NOISELESS CHANNEL

1. A channel is noiseless if it is lossless and deterministic.
2. A noiseless channel has only one element in each row and each column and the element is unity.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



BINARY SYMMETRIC CHANNEL

A binary symmetric channel has two inputs (x_1=0, x_2=1) and two outputs (y_1=0, y_2=1)

$$[P(Y|X)] = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$